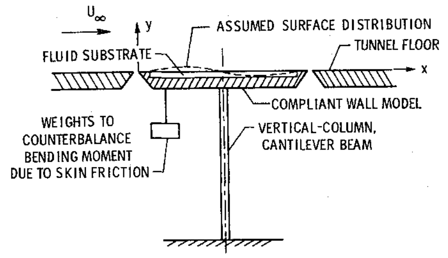


**Fig. 1 Simplified schematic of models used in Refs. 6-8.**

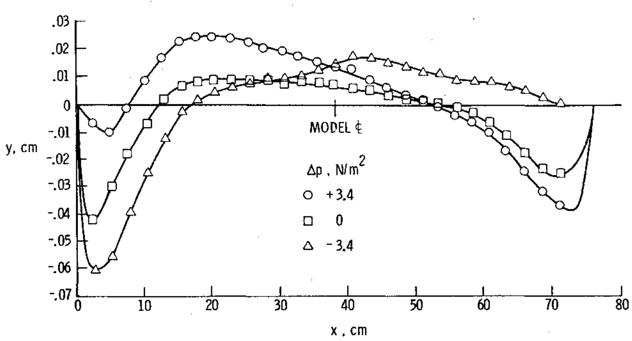


**Fig. 2 Surface motion measurements for compliant surface with water substrate.**

stantaneous surface angle over a spot 0.13 cm in diameter to within 0.002 of a degree. For the present study, only one detector was used and a time average over a long period of time was used at each $x$ position to determine the average surface angle. The original wind-off surface angles and then the wind-on surface angles were measured, and the differences between the two were integrated to obtain the flow induced change in surface position.

Surface position measurements on the PVC compliant skin model with water-filled substrate are presented in Fig. 2 for three longitudinal pressure gradients. Pressure gradients of these magnitudes were found in Refs. 6-8. References 6 and 7 modified the upper tunnel wall to eliminate or reduce the gradients whereas Ref. 8 corrected the drag measurements by calculating the bending moment induced by the variation in the surface pressures. In the present study, the gradients were obtained by moving the tunnel side walls $\pm 0.318$ cm from the mean zero pressure gradient position. These changes produced a 1.1% variation in the stream velocity with a 2.2% variation in static pressure. The corresponding changes in the static pressure ($\Delta p$) over the 76.2 cm long model was approximately $\pm 3.4$ N/m$^2$.

The large effect of only small pressure gradients on the formation of standing waves on the compliant wall surface is evident from Fig. 2. For the nearly zero pressure gradient the surface protrudes outward by approximately 0.009 cm. The positive gradient creates a bulge over the upstream portion of the model whereas the negative gradient causes the bulge to shift to the downstream portion of the surface. The model fairing plate around the compliant surface causes the surface to dip over the first 7.6-16.5 cm for all three gradients; the surface dips over the last 23 cm for the zero and slightly positive gradients.

The water volume under each of the three waves in Fig. 2 was integrated to determine the bending moment caused by the transfer of the water mass. The slightly positive gradient shifts the center of gravity upstream of the model centerline and generates a 3 gm-m bending moment. The near zero and negative pressure gradients shift the model center of gravity downstream of the model centerline and generate 0.1 gm-m and 3.3 gm-m bending moments, respectively. These bending moments in each situation would be sufficient to significantly alter the drag reductions reported in Refs. 6-8 and hence compromise the validity of the data (e.g., the 3 gm-m bending moment for the positive gradient could have indicated an apparent 70% "drag reduction").

The present analyses and measurements suggest a plausible explanation for previously measured compliant wall drag reductions.[6-8] References 6-8, however, are not the only experiments in the literature to show successful compliant wall drag reductions; a number of experiments (e.g., Refs. 1-5, 9, 10) still exist which support the compliant wall as a possible drag reduction device. The amount of drag reduction that can be obtained with compliant walls, the surface motion required for compliant wall drag reduction, and the mechanism responsible for the drag reduction must be better understood before an evaluation of compliant walls as a potential turbulent skin friction reduction concept for subsonic aircraft is possible.

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# Windward Plane of Symmetry Laminar Cross-Flow Effects

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## Nomenclature

$A$ = constant in skin friction law [see Eq. (3)]; at zero angle of attack, $A = 0.2205$ for laminar flow, $A = 0.0128$ for turbulent flow
$C_f$ = local skin friction coefficient
$f''_w$ = nondimensional velocity gradient at the body surface
$h$ = static enthalpy
$h_2$ = length element or scale factor which characterizes spreading of the streamlines
$H_e$ = total enthalpy

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$k$ = cross-flow parameter of Moore, Ref. 6
$K_{c_f}$ = correction factor due to coss-flow
$M$ = Mach number
$n$ = exponent in skin friction law [Eq. (3)]; $n=1.0$ for laminar flow, $n=0.25$ fo- turbulent flow
$p$ = pressure
$p_2$ = $\frac{1}{2}\partial^2 p/\partial\phi^2$
$Pr$ = Prandtl number
$Re_\theta$ = Reynolds number based on momentum thickness $=\rho_e u_e \theta/\mu_e$
$Re_x$ = Reynolds number based on streamwise coordinate $\rho_e u_e x/\mu_e$
$R$ = transverse radius of curvature of body
$u_e$ = streamwise edge of boundary-layer velocity
$x$ = streamwise coordinate
$\alpha$ = angle of attack
$\beta$ = correction factor on winward plane of symmetry due to wall gradient variation, [Eqs. (6) and (8)]
$\theta$ = momentum thickness
$\Theta_c$ = cone semi-vertex angle
$\mu$ = viscosity
$\xi$ = factor accounting for boundary layer property variations
$\rho$ = density
$\phi$ = circumferential body angle
$\omega$ = cross-flow parameter of Agnone (Eq. 2)

*Subscripts*

$e$ = edge condition
$\infty$ = freestream condition
$0$ = value at zero angle of attack
$x$ = derivative with respect to $x$
$\omega=0$ = equivalent cone condition

THERE are four major effects on local heat transfer and skin friction due to angle of attack. The first is the streamline development and therefore the flow history and wetted length to any specific point. The second is the variation in the value of the edge properties and local pressure with angle of attack including the effects of entrainment and entropy gradient. The third is the divergence effect of the flow which is accounted for in integral boundary-layer analyses using the scale factor $h_2$. This provides the effect of the thinning of the boundary layer due to geometric spreading of the flow. The fourth effect is the change of slope of the velocity and temperature profiles at the vehicle surface caused by the local cross-flow within the boundary layer. This cross-flow has been neglected within the framework of the "small cross-flow" or "axisymmetric analog" approach usually taken in integral type formulations. Smith and Chang[1] developed a method to calculate the laminar heating on a sharp cone at angle of attack employing an integral technqiue. Reshotko[2] and Brunk[3] developed solutions for the laminar heating on the windward plane of symmetry of sharp cones at angle of attack. Reshotko's solution is rigorous while Brunk's is based on an integral solution.

Adams[4] evaluated the results of "effective cone" calculations as compared to a three-dimensional windward meridian analysis for laminar, transitional, and turbulent flow over a slender sharp cone. He concluded that application of the "effective cone" concept to the windward streamline laminar boundary layer resulted in a severe under-prediction of heat transfer and skin friction compared to the windward plane-of-symmetry analysis. A smaller cross-flow effect resulted for the turbulent boundary layer.

Agone[5] developed a cross-flow correction factor to the effective cone technique for the plane-of-symmetry of a body at angle of attack. Agnone's correction factor $K_{c_f}$ is given in the following form

$$K_{c_f}=\frac{C_f}{C_{f_{\omega=0}}}=\left[1\pm\left(\frac{n+1}{n+2}\right)\omega\right]^{n/(n+1)} \tag{1}$$

where the subscripts $\omega=0$ denotes zero cross-flow effect, and $\omega$ is a cross-flow parameter on the plane of symmetry

$$\omega=-\tfrac{1}{2}\left\{1-[1-8p_2/(\rho_e u_e^2 R_x^2)]^{1/2}\right\} \tag{2}$$

$n$ is the exponent in the skin friction law; $n=1$ for laminar flow, and $n=0.25$ for a power law exponent of 1/7 for the turbulent velocity profile. For these values of $n$ and a specified value of $\omega$, Eq. (1) predicts a higher correction factor for laminar flow than for turbulent, a partial explanation for the greater success of the effective cone technqiue in turbulent flow. In the present Note it will be shown that the correction factor calculated using Eq. (1) must be augmented further in laminar flow, or windward plane-of-symmetry corrections to the effective cone method will still underpredict three-dimensional theory.

Eq. (1) is derived by employing the skin friction coefficient given by a modified Prandtl skin friction law

$$C_f/2=\xi A/Re_\theta^n \tag{3}$$

where $\xi$ accounts for variable properties. Assuming a linear variation of viscosity with temperature, for laminar flow $\xi$ is equal to the Chapman-Rubesin constant. $\xi$ is often based on Eckert reference properties.

For all other conditions equal, Eq. (1) provides the correction factor that accounts for streamline divergence or convergence on the plane of symmetry. It does not account for the effect on skin friction due to the change in the streamwise velocity profile gradient at the wall arising from the cross-flow terms in the momentum and continuity equations. This change would be reflected by a change in the value of the skin friction "constant" $A$, which at zero angle of attack is equal to 0.2205 for laminar flow and 0.0128 for turbulent flow, (the latter for a 1/7 power law velocity profile).

Equation (1) is derived by assuming $A$ is constant as angle of attack varies upward from zero, which is not in general true. Employing Agnone's results for the momentum thickness, and retaining $A$ as a variable, $K_{c_f}$ is refined to be given by

$$\begin{aligned}K_{c_f}&=\frac{C_f}{C_{f_0}}\left[\left(\frac{\xi_0}{\xi}\right)^{1/n}\frac{Re_x}{Re_{x_0}}\right]^{n/(n+1)}\\&=\left(\frac{A}{A_0}\right)^{1/(n+1)}\left[1\pm\left(\frac{n+1}{n+2}\right)\omega\right]^{n/(n+1)}\end{aligned} \tag{4}$$

The ratio of local Reynolds number and variable property functions contained in this equation have been retained so that the reference skin friction coefficient is evaluated at zero angle of attack, and not on an equivalent cone. Writing $K_{c_f}$ as a function of the nondimensional velocity gradient at the wall with and without cross-flow in laminar flow

$$K_{c_f}=\frac{f''_w}{f''_{w_0}}=\beta[1\pm k]^{1/2} \tag{5}$$

where

$$\beta=\left(\frac{A}{A_0}\right)^{1/2} \tag{6}$$

and $k$ is the circumferential cross-flow parameter as introduced by Moore.[6] $k$ is related to $\omega$ by

$$k=\tfrac{2}{3}\omega \tag{7}$$

Reshotko[2] and Brunk[3] provide numerical results for $f''_w$ on the windward plane of symmetry of a sharp cone. The streamline divergence and wall gradient variations due to

Table 1 Wall velocity gradient $f''_w$ and correction factor $\beta$

| $h_w/H_e$ | $H_e/h_e$ | $k$ | $f''_w$ | $\beta$ |
|---|---|---|---|---|
| 0.0 | 1.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.4330 | 1.0308 |
| | | 1.2 | 0.5143 | 1.0441 |
| | 3.5 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.4598 | 1.0946 |
| | | 1.2 | 0.5569 | 1.1306 |
| | 6.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.4815 | 1.1462 |
| | | 1.2 | 0.5898 | 1.1974 |
| 0.5 | 1.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.4468 | 1.0636 |
| | | 1.2 | 0.5367 | 1.0896 |
| | 3.5 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.4944 | 1.1769 |
| | | 1.2 | 0.6092 | 1.2367 |
| | 6.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.6 | 0.5291 | 1.2595 |
| | | 1.2 | 0.6594 | 1.3387 |
| 1.0 | 1.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.4 | 0.4215 | 1.0727 |
| | | 0.8 | 0.4935 | 1.1076 |
| | | 1.2 | 0.5559 | 1.1285 |
| | 2.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.4 | 0.4436 | 1.1289 |
| | | 0.8 | 0.5278 | 1.1846 |
| | | 1.2 | 0.5995 | 1.2171 |
| | 3.5 | 0.0 | 0.3321 | 1.0000 |
| | | 0.4 | 0.4704 | 1.1971 |
| | | 0.8 | 0.5675 | 1.2737 |
| | | 1.2 | 0.6487 | 1.3169 |
| | 6.0 | 0.0 | 0.3321 | 1.0000 |
| | | 0.4 | 0.5051 | 1.2580 |
| | | 0.8 | 0.6172 | 1.3852 |
| | | 1.2 | 0.7096 | 1.4406 |

cross-flow are contained implicitly in $f''_w$. Using Eq. (5), $\beta$ may be obtained from these results, which are given as a function of $k$, wall to total temperature ratio, and local Mach number. $\beta$ is the increase or decrease in skin friction coefficient (and for $Pr = 1.0$, the Stanton number) that occurs on the plane of symmetry of a sharp cone in laminar flow due only to the change in the velocity and temperature gradients at the wall created by the cross-flow gradient of velocity in the plane of symmetry. The values of $f''_w$ calculated by Reshotko are given in Table 1 as a function of $k$, $h_w/H_e$, and $H_e/h_e$. Using $f'_{w_0} = 0.3321$ and Eq. (5), the values of the correction factor associated with the change in velocity gradient at the wall with angle of attack may be obtained, and is also contained in Table 1. Note that the effect of the wall gradient on the total correction factor $K_{cf}$ is very significant under certain wall and edge enthalpy conditions. To neglect it by employing only the divergence effect, Eq. (1), could seriously underpredict the laminar skin friction.

The integral analysis of Brunk[3] provides numerical values of $f''_w$ that differ by less than 1% from Reshotko's exact values. By assuming appropriate temperature and velocity profiles, and employing these in the streamwise and circumferential integral equations in the windward plane of symmetry, Brunk provided simultaneous algebraic equations for the solution of the surface velocity gradients. Although presenting numerical comparisons with Reshotko's results, Brunk did not provide an explicit solution for $f''_w$. By solving the simultaneous equations obtained by Brunk, we find the following equation describing the total correction factor for laminar flow on the windward plane of symmetry

$$K_{c_f} = \beta[1+k]^{1/2} = \Big[0.4811 + 0.2864k + \Big\{0.2693 + 0.7408k + 0.5903k^2 + k(1+1.5k)\Big(0.1572\frac{H_e}{h_e} + 0.4168\frac{h_w}{h_e}\Big)\Big\}^{1/2}\Big]^{1/2} \tag{8}$$

If the term containing the enthalpy ratios in Eq. (8) is neglected, the solution essentially reduces to Eq. (5) with $\beta = 1.0$. The enthalpy ratios in the solution represent the effect of the coupling between the streamwise and circumferential momentum equations. When the terms which provide this coupling in the governing differential equations are dropped and the nondimensional circumferential velocity derivative and streamwise velocity are assumed equal, the solution provides only the divergence effects.

Figure 1 shows the variation of $K_{cf}$ with angle of attack on the windward meridian of a 7.5° cone at Mach 3.1, calculated using the analytical results of Moore.[6] Using only the streamline divergence correction factor, with $\beta = 1.0$, accounts for only about one half the increase in skin friction at these conditions. Including the effects of wall gradient variations brings the correction factor into agreement with Moore's calculations.

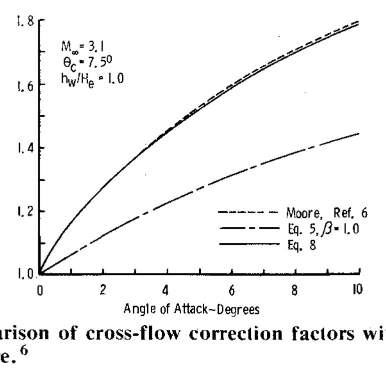


**Fig. 1 Comparison of cross-flow correction factors with analytical results of Moore.[6]**

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# Stability of a Gyrostat Satellite with Flexible Appendages


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## Introduction

THE stability of artificial satellites with flexible appendages has received much attention in recent years. The inclusion of distributed elastic parts in modeling satellites yields a hybrid dynamical system, that is, one which is described by both ordinary and partial differential equations.


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